Distilling a Greenberger-Horne-Zeilinger state from an arbitrary pure state of three qubits

Li-Xiang Cen[1]*and Shun-Jin Wang[1,2]

1. Department of Modern Physics, Lanzhou University, Lanzhou 730000, P.R.China

2. Institute of Modern Physics, Southwest Jiaotong University, Chengdu 610031, P.R.China

Abstract

We present a general algorithm to achieve local operators which can produce the GHZ state for an arbitrary given three-qubit state. Thus the distillation process of the state can be realized optimally. The algorithm is shown to be sufficient for the three-qubit state on account of the fact that any state for which this distillation algorithm is invalid cannot be distilled to the GHZ state by any local actions. Moreover, an analytical result of distillation operations is achieved for the general state of three qubits.

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Entanglement manipulation is an important issue in the studies of quantum information theory. On the one hand, it is related to the basic problem of which tasks one can accomplish with a given resource of entanglement [1]-[3]. On the other hand, since most applications of quantum information theory require the maximally entangled state for faithfully transmission of quantum data, it is necessary to develop the special technique of entanglement manipulation which uses local quantum operations and classical communication to purify impure entangled states [2, 4].

There has been extensive work on entanglement manipulation. For the case of two-qubit systems, the Procrustean method [2] provides local operations to obtain the maximally entangled state from a partly entangled pure state. The general theory of entanglement transformations for pure states of the bipartite system has also been proposed [5]-[7]. As far as tripartite states [8]-[11] are concerned, researches have shown that the GHZ state [12] is the maximally entangled state which violates Bell inequalities maximally and maximize the mutual information of local measurements [13]. Hence, it is desirable to propose a general algorithm to carry out the local operations to distill the GHZ state from a given tripartite state. In this paper we present such an algorithm. Using this algorithm, we obtain an analytical result of the distillation operations for the general state of three qubits. As far as we know, it is the first time for the result of this type to be achieved.

What we address is to transform a tripartite state $|\psi_{ABC}\rangle$ of three qubits A, B and C into the state $|\psi_{GHZ}\rangle \equiv \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ by local actions

$$\frac{1}{N^{1/2}} T_A \otimes T_B \otimes T_C |\psi_{ABC}\rangle = |\psi_{GHZ}\rangle, \qquad (1)$$

where $N = \langle\psi_{ABC}|T_A^\dagger T_A \otimes T_B^\dagger T_B \otimes T_C^\dagger T_C|\psi_{ABC}\rangle$ is the probability that the manipulation succeeded. Once the operators T_A, T_B and T_C are worked out, one can construct the simplest local operations—generalized measurements or actions of local filters on qubits A, B, and C—to implement the distillation process optimally.

We first give a special representation for the three-qubit state $|\psi_{ABC}\rangle$, which will be useful in our distillation method. The representation, which we call "Wootters' representation" here, is introduced according to the following idea. Suppose, the tripartite state $|\psi_{ABC}\rangle$ considered has a form

$$|\psi_{ABC}\rangle = \sum_{i=0}^{1} |i_A\rangle|\varphi_i^{\bar{A}}\rangle, \qquad (2)$$

where $\{|0_A\rangle, |1_A\rangle\}$ are the standard basis of qubit A, and $|\varphi_i^{\bar{A}}\rangle$s are states of the ensemble which realizes the reduced density matrix ρ_{BC} of qubits B and C. Note that $|\varphi_i^{\bar{A}}\rangle$ is subnormalized, namely, the squared length $\langle\varphi_i^{\bar{A}}|\varphi_i^{\bar{A}}\rangle$ is equal to the probability of $|\varphi_i^{\bar{A}}\rangle$ in the ensemble. We then can get a kind of representations of $|\psi_{ABC}\rangle$ by different choices of the basis of qubit A

$$|\psi_{ABC}\rangle = \sum_{i=0}^{1} |i'_A\rangle|\phi_i^{\bar{A}}\rangle \qquad (3)$$

with

$$|i'_A\rangle = \hat{V}_A^{\intercal}|i_A\rangle = \sum_{j=0}^{1}(V_A)_{ij}|j_A\rangle,\ |\phi_i^{\bar{A}}\rangle = \sum_{j=0}^{1}(V_A^*)_{ij}|\varphi_j^{\bar{A}}\rangle. \qquad (4)$$

Here $i = 0, 1$ and the transposition is taken in the standard basis $\{|i_A\rangle\}$ (it is included in order to

*Electronic address: gontp@spin.lzu.edu.cn

be consistent with previous papers, e.g., see Ref. [14]). The matrix V_A is a representation of the unitary operator $\hat{V}_A$ in the standard basis. Introduce the time reversal operation for two-qubit states with $|\tilde{\phi}\rangle = \sigma_2 \otimes \sigma_2 |\phi^*\rangle$, and define the symmetric matrix $\tau_{\phi^{\bar{A}}}$ for $\{|\phi_i^{\bar{A}}\rangle\}$ with

$$\tau^{ij}_{\phi^{\bar{A}}} = \langle \phi_i^{\bar{A}} | \tilde{\phi}_j^{\bar{A}} \rangle, \quad i = 0, 1. \tag{5}$$

Then according to the studies of Wootters [14], there exists a representation

$$|\psi_{ABC}\rangle = \sum_{i=0}^{1} |A_i\rangle |x_i^{\bar{A}}\rangle \tag{6}$$

with

$$|A_i\rangle = \hat{U}_A^{\intercal} |i_A\rangle = \sum_{j=0}^{1} (U_A)_{ij} |j_A\rangle, \; |x_i^{\bar{A}}\rangle = \sum_{j=0}^{1} (U_A^*)_{ij} |\varphi_j^{\bar{A}}\rangle \tag{7}$$

such that

$$\tau^{ij}_{x^{\bar{A}}} = (U_A \tau_{\varphi^{\bar{A}}} U_A^{\intercal})_{ij} = \delta_{ij} \pi_i^{\bar{A}}, \quad i, j = 0, 1. \tag{8}$$

Here the parameters $\pi_i^{\bar{A}}$s, which satisfy $\pi_0^{\bar{A}} \geq \pi_1^{\bar{A}} \geq 0$, are the square roots of the eigenvalues of the Hermitian matrix $\tau_{\phi^{\bar{A}}} \tau^*_{\phi^{\bar{A}}}$ [15]. The difference of them defines a "concurrence"[14, 16] which provides a measure of entanglement for the two-qubit state ρ_{BC}.

Similarly, we can write down two other Wootters' representations of the state $|\psi_{ABC}\rangle$

$$|\psi_{ABC}\rangle = \sum_{i=0}^{1} |i_B\rangle |\varphi_i^{\bar{B}}\rangle = \sum_{i=0}^{1} |B_i\rangle |x_i^{\bar{B}}\rangle,$$

$$|\psi_{ABC}\rangle = \sum_{i=0}^{1} |i_C\rangle |\varphi_i^{\bar{C}}\rangle = \sum_{i=0}^{1} |C_i\rangle |x_i^{\bar{C}}\rangle \tag{9}$$

with relations

$$|R_i\rangle = \hat{U}_R^{\intercal} |i_R\rangle = \sum_{j=0}^{1} (U_R)_{ij} |j_R\rangle, \; |x_i^{\bar{R}}\rangle = \sum_{j=0}^{1} (U_R^*)_{ij} |\varphi_j^{\bar{R}}\rangle,$$

$$\langle x_i^{\bar{R}} | \tilde{x}_j^{\bar{R}} \rangle = \delta_{ij} \pi_i^{\bar{R}}, \qquad R = B, C, \quad i = 0, 1. \tag{10}$$

Our distillation algorithm makes use of a set of local operations

$$f_R = (\frac{\pi_1^{\bar{R}}}{\pi_0^{\bar{R}}})^{1/2} |R_0\rangle\langle R_0| + |R_1\rangle\langle R_1|$$

$$= U_R^{\intercal} [(\frac{\pi_1^{\bar{R}}}{\pi_0^{\bar{R}}})^{1/2} |0_R\rangle\langle 0_R| + |1_R\rangle\langle 1_R|] U_R^*, \; R = A, B, C. \tag{11}$$

Note that these operations require $\pi_1^{\bar{R}} \neq 0$ since for the case of $\pi_1^{\bar{R}} = 0$, f_R becomes a projection operator of the pure state of qubit R and it shall disentangle the tripartite state. In fact, the states with $\pi_1^{\bar{R}} = 0$ cannot be distilled to the GHZ state by any local actions and classical communication acting individually on them. Further discussions shall be presented later in this paper.

Direct observation can be found that the action f_R on the qubit R ($R = A, B, C$) causes the reduced matrix of the two other qubits to be separable. For example, consider the operation f_A on the qubit A. According to the Wootters' representation (6), after the operation, one shall acquire a state (note that such a state is obtained probalistically) with $\pi_0^{\bar{A}} = \pi_1^{\bar{A}}$. This means that the reduced matrix ρ_{BC} has a zero concurrence, thus is nonentangled.

Theorem 1: A tripartite entangled state of three qubits $|\psi_{ABC}\rangle$ which has Wootters' representations with $\pi_1^{\bar{R}} \neq 0$ ($R = A, B, C$) can be transformed to the generalized GHZ state by the local operations

$$f_A \otimes f_B \otimes f_C |\psi_{ABC}\rangle, \tag{12}$$

where the operators f_A, f_B, and f_C are given by Eq. (11). Note that a generalized GHZ state $|\psi_{GGHZ}\rangle$ has the following form

$$|\psi_{GGHZ}\rangle = \alpha |000\rangle + \beta |111\rangle \tag{13}$$

with the parameters $0 \leq \alpha \leq \beta$ by a suitable choice of the local basis.

Proof: Consider the reduced density matrix ρ_{BC} of qubits B and C. After the local operation f_A, it becomes a separable state. Since the local operations f_B and f_C can not produce any entanglement for the nonentangled state of qubits B and C, it shall still be a separable state in the final outcome of the set of actions (12). According to the communicative property of the set of operators $\{f_A, f_B, f_C\}$, the same analysis is applicable to the reduced density of qubit pairs A-B and A-C. Thus, after the local operations (12), we arrive at a state which is pairwise separable. We now need to show that a pairwise separable state of three qubits is a generalized GHZ state.

Lemma 1: A tripartite entangled state of three qubits is pairwise separable if and only if it is a generalized GHZ state.

Proof: It is obvious that a generalized GHZ state (13) is pairwise separable. To prove the converse statement, we use the fact that an arbitrary tripartite state of three qubits can be written in the form [9]

$$|\psi_{ABC}\rangle = \lambda_0 |000\rangle + \lambda_1 e^{i\varphi} |100\rangle + \lambda_2 |101\rangle + \lambda_3 |110\rangle + \lambda_4 |111\rangle \tag{14}$$

by a suitable choice of local basis, where the coefficients λ_i are all real and non-negative and φ is a phase between 0 and π. This representation is a minimal description, with only five terms, for the three-qubit states. Now, suppose a tripartite entangled state $|\psi^s_{ABC}\rangle$ which

has a minimal representation with coefficients λ_i^s $(i = 0, \cdots, 4)$ and a phase φ^s is pairwise separable. Then, due to separability, there are relations $\pi_0^{\bar{R}} = \pi_1^{\bar{R}}$ $(R = A, B, C)$ for the Wootters' representations of $|\psi_{ABC}^s\rangle$. A straightforward calculation yields

$$\lambda_2^s \lambda_3^s - \lambda_1^s \lambda_4^s e^{i\varphi^s} = 0, \tag{15}$$

$$\lambda_0^s \lambda_2^s = 0, \tag{16}$$

$$\lambda_0^s \lambda_3^s = 0. \tag{17}$$

Noticing that $|\psi_{ABC}^s\rangle$ is a tripartite entangled state, i.e., it can not be written as a biseparable form and a product form of the three qubits, we then have the solutions $\lambda_1^s = \lambda_2^s = \lambda_3^s = 0$ for the above set of equations. Now the state $|\psi_{ABC}^s\rangle$ takes the form

$$|\psi_{ABC}^s\rangle = \lambda_0^s|000\rangle + \lambda_4^s|111\rangle. \tag{18}$$

This completes our proof of Lemma 1 and then Theorem 1.

Let $|\psi'_{ABC}\rangle$ denote the state of the outcome corresponding to the local operations of (12). According to Theorem 1, we have

$$|\psi'_{ABC}\rangle = \alpha|0'0'0'\rangle + \beta|1'1'1'\rangle. \tag{19}$$

The coefficients α, β and the local basis $\{|0'\rangle, |1'\rangle\}$ of the three qubits are completely determined by the initially given state $|\psi_{ABC}\rangle$. They can be directly calculated from Eqs. (11) and (12). Now to acquire the GHZ state one only needs to carry out one of the following actions (noticing that we have set $\alpha \leq \beta$)

$$f'_R = |0'_R\rangle\langle 0'_R| + \frac{\alpha}{\beta}|1'_R\rangle\langle 1'_R|, \ R = A, B, C \tag{20}$$

on the qubits A, B or C respectively.

The algorithm established above shall enable one to achieve local operators of the distillation actions for any pure tripartite state of three qubits. Before presenting the general result for such distillation actions, we give some discussions which our algorithm implies.

In general, to distill the GHZ state from a three-qubit state requires local operations on each of the three qubits. Nevertheless, there are exceptions. The simplest example is the generalized GHZ state for which one only needs to perform a local operation on any one of the three qubits. In detail, on which qubits the operations to be performed is determined by properties of the three reduced matrices ρ_{AB}, ρ_{AC}, and ρ_{BC}—whether they are separable or not. For instance, assume ρ_{AB} of the state $|\psi_{ABC}\rangle$ is separable. Then we have $\pi_0^{\bar{C}} = \pi_1^{\bar{C}}$ due to its separability. This results in that the local operator f_C of Eq. (11) becomes an identity operator. A noticeable example of this type is the slice state [17] which has the form

$$|\psi_{slice}\rangle = \lambda_0|000\rangle + \lambda_1|100\rangle + \lambda_4|111\rangle. \tag{21}$$

A simple calculation shows that the reduced matrices ρ_{AB} and ρ_{AC} of it are separable. Thus to achieve the GHZ state from $|\psi_{slice}\rangle$ one only needs to perform a local operation T_A on the qubit A.

Now we present an analytical result of distillation operators $T_A \otimes T_B \otimes T_C$ in (1) for the general pure three-qubit state. We still use the minimal representation (14) for the state $|\psi_{ABC}\rangle$. Direct calculations give the parameters of the state $|\psi_{ABC}\rangle$

$$\begin{aligned}
\pi_{0,1}^{\bar{A}} &= \sqrt{|\Delta|^2 + \lambda_0^2\lambda_4^2} \pm |\Delta|, \\
\pi_{0,1}^{\bar{B}} &= \lambda_0(\sqrt{\lambda_2^2 + \lambda_4^2} \pm \lambda_2), \\
\pi_{0,1}^{\bar{C}} &= \lambda_0(\sqrt{\lambda_3^2 + \lambda_4^2} \pm \lambda_3),
\end{aligned} \tag{22}$$

where '+' is for $\pi_0^{\bar{R}}$ and '−' for $\pi_1^{\bar{R}}$. The complex number Δ is defined as

$$\Delta \equiv \lambda_2\lambda_3 - \lambda_1\lambda_4 e^{i\varphi}. \tag{23}$$

The local operations which transform the state $|\psi_{ABC}\rangle$ into the GHZ state are given by

$$T_A \otimes T_B \otimes T_C = U'_A f_A \otimes U'_B f_B \otimes U'_C f'_C f_C, \tag{24}$$

where the operators are shown as follows:

f_R: f_R $(R = A, B, C)$ is given by Eq. (11). The unitary operator $\hat{U}_R$ in it comprises two parts

$$U_R = U_{R1} U_{R0}. \tag{25}$$

Here, $\hat{U}_{R0}$ is a unitary transformation which diagonalizes the Hermitian matrix $\tau_{\varphi^{\bar{R}}}\tau^*_{\varphi^{\bar{R}}}$. Direct calculations give that

$$U_{A0} = e^{-i\theta_2\sigma_2}e^{-i\theta_3\sigma_3}, U_{B0} = e^{-i\theta_B\sigma_2}, U_{C0} = e^{-i\theta_C\sigma_2}, \tag{26}$$

where

$$\theta_2 = \arctan\frac{|\Delta| - \sqrt{|\Delta|^2 + \lambda_0^2\lambda_4^2}}{\lambda_0\lambda_4}, \ \theta_3 = -\arctan\frac{|\Delta| + \mathrm{Re}\,\Delta}{\mathrm{Im}\,\Delta},$$
$$\theta_B = \arctan\frac{\lambda_2 - \sqrt{\lambda_2^2 + \lambda_4^2}}{\lambda_4}, \ \theta_C = \arctan\frac{\lambda_3 - \sqrt{\lambda_3^2 + \lambda_4^2}}{\lambda_4}. \tag{27}$$

The operator

$$U_{R1} = i|1_R\rangle\langle 1_R| + |0_R\rangle\langle 0_R| \tag{28}$$

is included so that the diagonal elements of $U_R\tau_{\varphi^{\bar{R}}}U_R^{\intercal}$ become real and non-negative.

f'_R: The operator f'_R $(R = A, B, \text{ or } C)$ of (20) transforms the generalized GHZ state into the GHZ state. It is equivalent to perform any one of them. In equation (24) we have chosen the

operation f'_C acting on the qubit C. To achieve f'_R requires the detailed knowledge of parameters and local basis of the generalized GHZ state (19). After some hard but straightforward calculations, we obtain the ratio of the two coefficients

$$\frac{\alpha}{\beta} = \left[\frac{(\lambda_2^2+\lambda_4^2)(\lambda_3^2+\lambda_4^2)}{|\Delta|^2+\lambda_0^2\lambda_4^2}\right]^{1/2} \tag{29}$$

and the local basis of the qubit C

$$|0'_C\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_C+\sin\theta_C)|0_C\rangle+(\cos\theta_C-\sin\theta_C)|1_C\rangle],$$
$$|1'_C\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_C-\sin\theta_C)|0_C\rangle-(\cos\theta_C+\sin\theta_C)|1_C\rangle]. \tag{30}$$

Here we assume $(\lambda_2^2+\lambda_4^2)(\lambda_3^2+\lambda_4^2) \leq |\Delta|^2+\lambda_0^2\lambda_4^2$, and the parameter Δ is given in Eq. (23) and θ_C in (27).

U'_R: The final local unitary transformations

$$U'_R = |0_R\rangle\langle 0'_R| + |1_R\rangle\langle 1'_R| \quad R = A, B, C \tag{31}$$

are included to revert the local basis $\{|0'_R\rangle, |1'_R\rangle\}$ of generalized GHZ state (19) to the initial standard basis $\{|0_R\rangle, |1_R\rangle\}$ of the state $|\psi_{ABC}\rangle$. $\{|0'_C\rangle, |1'_C\rangle\}$ have been given in Eq. (30). Now we present the local basis of qubits A and B

$$|0'_A\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_2+\sin\theta_2)e^{-i\theta_3}|0_A\rangle+(\cos\theta_2-\sin\theta_2)e^{i\theta_3}|1_A\rangle],$$
$$|1'_A\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_2-\sin\theta_2)e^{-i\theta_3}|0_A\rangle-(\cos\theta_2+\sin\theta_2)e^{i\theta_3}|1_A\rangle],$$
$$|0'_B\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_B+\sin\theta_B)|0_B\rangle+(\cos\theta_B-\sin\theta_B)|1_B\rangle],$$
$$|1'_B\rangle = \frac{\sqrt{2}i}{2}[(\cos\theta_B-\sin\theta_B)|0_B\rangle-(\cos\theta_B+\sin\theta_B)|1_B\rangle], \tag{32}$$

where θ_2, θ_3, and θ_B are given by (27).

There remains one case to consider, namely, the states with parameters $\pi_1^{\bar{R}} = 0$ $(R = A, B, C)$ for their Wootters' representations. For these states our distillation algorithm is not valid any more. In fact, as far as the tripartite entangled state is concerned, the three relations $\pi_1^{\bar{R}} = 0$ for $R = A, B$ and C are equivalent. For example, the condition $\pi_1^{\bar{A}} = 0$ implies that $\pi_1^{\bar{B}} = \pi_1^{\bar{C}} = 0$. This can be seen from Eq. (22). Assume $\pi_1^{\bar{A}} = 0$ for the state $|\psi_{ABC}\rangle$. Since $\lambda_0 \neq 0$ (otherwise the state $|\psi_{ABC}\rangle$ will be biseparable), the relation $\pi_1^{\bar{A}} = 0$ leads to $\lambda_4 = 0$, and then $\pi_1^{\bar{B}} = \pi_1^{\bar{C}} = 0$. The state $|\psi_{ABC}\rangle$ now has the form

$$|\psi_{ABC}\rangle = \lambda_0|000\rangle+\lambda_1 e^{i\varphi}|100\rangle+\lambda_2|101\rangle+\lambda_3|110\rangle. \tag{33}$$

This state is the so-called "W-class state" [10] which could not be distilled to the GHZ state by any local actions. It can be understood in the following way. Since, the ratio $\pi_1^{\bar{A}}/\pi_0^{\bar{A}}$ is a constant under the invertible local operations of qubits B and C (see Ref. [18, 19]), the relation $\pi_1^{\bar{A}} = \pi_1^{\bar{B}} = \pi_1^{\bar{C}} = 0$ shall be retained under the actions $T_B \otimes T_C$. Similar analysis gives that this relation shall also hold under the action of T_A. Thus we can conclude that local operations $T_A \otimes T_B \otimes T_C$ shall take a W-class state to another W-class state, so that the GHZ state will never be produced.

In summary, we have presented an algorithm to distill the GHZ state from a single copy of the three-qubit state. It enable one to achieve directly the operators of the distillation operations, thus the distillation process can be realized optimally. We then apply our distillation algorithm to the general state of three qubits and obtain an analytical result of operations for such a process. Finally, we show that the state for which our distillation algorithm is not valid cannot be distilled to the GHZ state by any local operations. Thus the distillation algorithm we presented is sufficient for the tripartite state of three qubits.

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